6.2 Lesson

Name________________

Essential Question____

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What You Will Learn

- Find *n*th roots.
- Evaluate expressions with rational exponents.
- Solve real-life problems involving rational exponents.

Finding *n*th Roots

You can extend the concept of a square root to other types of roots. For example, 2 is a cube root of 8 because $2^3 = 8$, and 3 is a fourth root of 81 because $3^4 = 81$. In general, for an integer n greater than 1, if $b^n = a$, then b is an **nth root of a**. An nth root of a is written as $\sqrt[n]{a}$, where the expression $\sqrt[n]{a}$ is called a **radical** and n is the **index** of the radical.

You can also write an nth root of a as a power of a. If you assume the Power of a Power Property applies to rational exponents, then the following is true.

$$(a^{1/2})^2 = a^{(1/2)\cdot 2} = a^1 = a$$
$$(a^{1/3})^3 = a^{(1/3)\cdot 3} = a^1 = a$$
$$(a^{1/4})^4 = a^{(1/4)\cdot 4} = a^1 = a$$

Because $a^{1/2}$ is a number whose square is a, you can write $\sqrt{a} = a^{1/2}$. Similarly, $\sqrt[3]{a} = a^{1/3}$ and $\sqrt[4]{a} = a^{1/4}$. In general, $\sqrt[n]{a} = a^{1/n}$ for any integer n greater than 1.

Core Concept

READING

$\pm\sqrt[n]{a}$ represents both the positive and negative nth roots of a.

Real *n*th Roots of *a*

Let n be an integer greater than 1, and let a be a real number.

- If n is odd, then a has one real nth root: $\sqrt[n]{a} = a^{1/n}$
- If n is even and $a > 0$, then a has two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$
- If n is even and $a = 0$, then a has one real nth root: $\sqrt[n]{0} = 0$
- If n is even and $a < 0$, then a has no real nth roots.

EXAMPLE 1 Finding *n*th Roots

Find the indicated real nth root(s) of a.

a. $n = 3, a = -27$

b. $n = 4, a = 16$

Monitoring Progress

Find the indicated real nth root(s) of a.

1. $n = 3, a = -125$

2. $n = 6, a = 64$

Evaluating Expressions with Rational Exponents

Recall that the radical $\sqrt{a}$ indicates the positive square root of a. Similarly, an nth root of a, $\sqrt[n]{a}$, with an *even* index indicates the positive nth root of a.

REMEMBER

The expression under the radical sign is the radicand.

EXAMPLE 2 Evaluating *n*th Root Expressions

Evaluate each expression.

a. $\sqrt[3]{-8}$

b. $-\sqrt[3]{8}$

c. $16^{1/4}$

d. $(-16)^{1/4}$

STUDY TIP

You can rewrite $27^{2/3}$ as $27^{(1/3)\cdot 2}$ and then use the Power of a Power Property to show that

$$27^{(1/3)\cdot 2} = (27^{1/3})^2.$$

Core Concept

Rational Exponents

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

Algebra $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$

Numbers $27^{2/3} = (27^{1/3})^2 = (\sqrt[3]{27})^2$

EXAMPLE 3 Evaluating Expressions with Rational Exponents

Evaluate (a) $16^{3/4}$ and (b) $27^{4/3}$.

Monitoring Progress

Evaluate the expression.

3. $\sqrt[3]{-125}$ **4.** $(-64)^{2/3}$ **5.** $9^{5/2}$ **6.** $256^{3/4}$

Solving Real-Life Problems

EXAMPLE 4 Solving a Real-Life Problem

Volume = 113 cubic feet

The radius r of a sphere is given by the equation $r = \left(\frac{3V}{4\pi}\right)^{1/3}$, where V is the volume of the sphere. Find the radius of the beach ball to the nearest foot. Use 3.14 for π.